THE THICK WALL METHOD OF INVESTIGATING HEAT TRANSFER IN CROSS FLOW OVER A TUBE

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The heat transfer coefficient is found from the measured temperature distribution over the surface of a tube. An analytical and a net method are examined in relation to the wall of a tube with and without internal heat sources.

A method has been developed [2] for determining the local coefficient of heat transfer between a tube and a cross flow of liquid from the temperatures measured on both faces of the annular cross section. In this method the local coefficient is given by

$$\alpha_l = -\frac{\lambda}{\theta}\left(\frac{\partial t}{\partial r}\right)_{r_2}. \tag{1}$$

The temperature gradient in (1) is determined from the temperature field, which is found by integrating the differential equation of heat conduction for a given temperature distribution at the boundaries of the section.

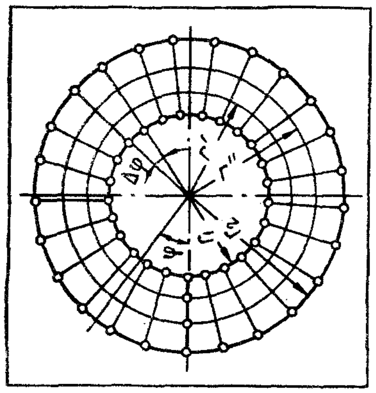

Fig. 1. Cross section of tube.

The average heat transfer coefficient over a portion of the tube from 0 to φ (Fig. 1) is given by

$$\alpha = \frac{1}{\varphi\theta_m}\int_0^{\varphi}\alpha_l\theta\, d\varphi. \tag{2}$$

In the analytical method of reducing the experimental data, the temperature gradient is expressed as a slowly converging series, and it is therefore more convenient first to calculate the average heat transfer coefficient

$$\alpha = -\frac{\lambda}{\theta_m\varphi}\int_0^{\varphi}\left(\frac{\partial t}{\partial r}\right)_{r_2} d\varphi \tag{3}$$

and then determine the local coefficient graphically or analytically. The series in terms of which the integral in (3) is expressed converges considerable more rapidly than that for calculating the temperature gradient.

We will now examine the analytical method based on (3), and also a net method, in which the coefficient is determined from (1) and (2).

In the analytical method, for tubes without internal heat release, the integral in (3) is determined with the aid of the differential equation

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2} = 0, \tag{4}$$

whose solution must fit the boundary conditions

$$\begin{gathered} t(r, \varphi) = t(r, \varphi + 2\pi), \\ t(r_1, \varphi) = \xi(\varphi), \quad t(r_2, \varphi) = \zeta(\varphi). \end{gathered} \tag{5}$$

Solution of (4) with boundary conditions (5) by the Fourier method yields

$$\begin{aligned} t = A_0 + B_0 \ln r + \sum_{n=1}^{\infty} [(A_n r^n + B_n r^{-n}) \cos n\varphi + \\ + (C_n r^n + D_n r^{-n}) \sin n\varphi], \end{aligned} \tag{6}$$

where n = 1, 2, 3, ...

The coefficients in (6) are determined from the temperature distribution over the tube surfaces:

$$A_0 = \frac{M_0 \ln r_2 - N_0 \ln r_1}{2\ln (r_2/r_1)}, \quad B_0 = \frac{N_0 - M_0}{2\ln (r_2/r_1)},$$
$$A_n = \frac{M_n r_2^{-n} - N_n r_1^{-n}}{r_1^n r_2^{-n} - r_1^{-n} r_2^n}, \quad B_n = \frac{N_n r_1^n - M_n r_2^n}{r_1^n r_2^{-n} - r_1^{-n} r_2^n},$$
$$C_n = \frac{M_n^* r_2^{-n} - N_n^* r_1^{-n}}{r_1^n r_2^{-n} - r_1^{-n} r_2^n}, \quad D_n = \frac{N_n^* r_1^n - M_n^* r_2^n}{r_1^n r_2^{-n} - r_1^{-n} r_2^n}. \tag{7}$$

Here

$$M_0 = \frac{1}{\pi}\int_0^{2\pi} \xi(\varphi)\, d\varphi, \quad N_0 = \frac{1}{\pi}\int_0^{2\pi} \zeta(\varphi)\, d\varphi,$$
$$M_n = \frac{1}{\pi}\int_0^{2\pi} \xi(\varphi)\cos n\varphi\, d\varphi, \quad N_n = \frac{1}{\pi}\int_0^{2\pi} \zeta(\varphi)\cos n\varphi\, d\varphi,$$
$$M_n^* = \frac{1}{\pi}\int_0^{2\pi} \xi(\varphi)\sin n\varphi\, d\varphi, \quad N_n^* = \frac{1}{\pi}\int_0^{2\pi} \zeta(\varphi)\sin n\varphi\, d\varphi.$$

The integral in (3) is determined using (6):

$$\int_0^{\varphi} \left(\frac{\partial t}{\partial r}\right)_{r_2} d\varphi = \frac{B_0}{r_2}\varphi + \sum_{n=1}^{\infty} [(A_n r_2^{n-1} - B_n r_2^{-n-1})\sin n\varphi - (C_n r_2^{n-1} - D_n r_2^{-n-1})(\cos n\varphi - 1)]. \tag{8}$$

For a tube with heat release in the walls*, the differential equation has the form:

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2} + \frac{q_v}{\lambda} = 0. \tag{9}$$

If the output of the internal heat sources does not depend on temperature, then the substitution

$$T = t + \frac{q_v}{\lambda}\frac{r^2}{4} \tag{10}$$

reduces (9) to the same form as (4):

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} = 0. \tag{11}$$

The boundary conditions have the form

$$T(r, \varphi) = T(r, \varphi + 2\pi),$$
$$T(r_1, \varphi) = \xi + \frac{q_v}{\lambda}\frac{r_1^2}{4} = F(\varphi),$$
$$T(r_2, \varphi) = \zeta(\varphi) + \frac{q_v}{\lambda}\frac{r_2^2}{4} = \Phi(\varphi).$$

The integral of (11) coincides with (6), and the coefficients in it are determined from (7). In determining the parameters M_0, N_0, M_n, N_n, M_n^* and N_n^*, the functions $\xi(\varphi)$ and $\zeta(\varphi)$ should be replaced by $F(\varphi)$ and $\Phi(\varphi)$.

In this case the final formula for determining the integral in (3) has the form:

*This type of problem arises in investigating heat transfer in the presence of a magnetic field or when the tube is heated by an electric current.

$$\int_0^{\varphi}\left(\frac{\partial t}{\partial r}\right)_{r_2} d\varphi=\left(\frac{B_0}{r_2}-\frac{q_v}{\lambda}\,\frac{r_2}{2}\right)\varphi+\sum_{n=1}^{\infty}\left[(A_n r_2^{n-1}-B_n r_2^{-n-1})\sin n\varphi-\right.$$
$$\left.-(C_n r_2^{n-1}-D_n r_2^{-n-1})(\cos n\varphi-1)\right]. \tag{12}$$

Equation (9) can also be solved if it is assumed that the output of the internal heat sources does depend on temperature. In this case it may be written as

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\,\frac{\partial t}{\partial r}+\frac{1}{r^2}\,\frac{\partial^2 t}{\partial\varphi^2}+a+bt=0.$$

The substitution $T = a + bt$ reduces it to the form:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\,\frac{\partial T}{\partial r}+\frac{1}{r^2}\,\frac{\partial^2 T}{\partial\varphi^2}+bT=0. \tag{13}$$

Here the boundary conditions are

$$T(r,\ \varphi)=T(r,\ \varphi+2\pi),$$
$$T(r_1,\ \varphi)=a+b\,\xi(\varphi)=K(\varphi),\quad T(r_2,\ \varphi)=a+b\,\zeta(\varphi)=L(\varphi).$$

The integral of (13), obtained by the Fourier method, has the form

$$T=A_0J_0(r\sqrt{b})+B_0Y_0(r\sqrt{b})+\sum_{n=1}^{\infty}\{[A_nJ_n(r\sqrt{b})+$$
$$+B_nY_n(r\sqrt{b})]\cos n\varphi+[C_nJ_n(r\sqrt{b})+D_nY_n(r\sqrt{b})]\sin n\varphi\}. \tag{14}$$

The coefficients in (14) may be determined from the boundary conditions:

$$A_0=\frac{1}{2}\,\frac{M_0Y_0(r_2\sqrt{b})-N_0Y_0(r_1\sqrt{b})}{J_0(r_1\sqrt{b})Y_0(r_2\sqrt{b})-J_0(r_2\sqrt{b})Y_0(r_1\sqrt{b})},$$

$$B_0=\frac{1}{2}\,\frac{N_0J_0(r_1\sqrt{b})-M_0J_0(r_2\sqrt{b})}{J_0(r_1\sqrt{b})Y_0(r_2\sqrt{b})-J_0(r_2\sqrt{b})Y_0(r_1\sqrt{b})},$$

$$A_n=\frac{M_nY_n(r_2\sqrt{b})-N_nY_n(r_1\sqrt{b})}{J_n(r_1\sqrt{b})Y_n(r_2\sqrt{b})-J_n(r_2\sqrt{b})Y_n(r_1\sqrt{b})},$$

$$B_n=\frac{N_nJ_n(r_1\sqrt{b})-M_nJ_n(r_2\sqrt{b})}{J_n(r_1\sqrt{b})Y_n(r_2\sqrt{b})-J_n(r_2\sqrt{b})Y_n(r_1\sqrt{b})},$$

$$C_n=\frac{M_n^*Y_n(r_2\sqrt{b})-N_n^*Y_n(r_1\sqrt{b})}{J_n(r_1\sqrt{b})Y_n(r_2\sqrt{b})-J_n(r_2\sqrt{b})Y_n(r_1\sqrt{b})},$$

$$D_n=\frac{N_n^*J_n(r_1\sqrt{b})-M_n^*J_n(r_2\sqrt{b})}{J_n(r_1\sqrt{b})Y_n(r_2\sqrt{b})-J_n(r_2\sqrt{b})Y_n(r_1\sqrt{b})}.$$

In calculating the parameters M_0, N_0, M_n, N_n, M_n^*, N_n^* from the equations examined above, $K(\varphi)$ and $L(\varphi)$ should be used in place of $\xi(\varphi)$ and $\zeta(\varphi)$.

Using (14), we can find the parameter required to calculate α from (3):

$$\int_0^{\varphi}\left(\frac{\partial t}{\partial r}\right)_{r_2} d\varphi=-\frac{1}{\sqrt{b}}\left[A_0J_1(r_2\sqrt{b})+B_0Y_1(r_2\sqrt{b})\right]\varphi+$$
$$+\frac{1}{nb}\sum_{n=1}^{\infty}\left\{\left[A_n\frac{n}{r_2}J_n(r_2\sqrt{b})-A_n\sqrt{b}\,J_{n+1}(r_2\sqrt{b})+\right.\right.$$
$$\left.+B_n\frac{n}{r_2}Y_n(r_2\sqrt{b})-B_n\sqrt{b}\,Y_{n+1}(r_2\sqrt{b})\right]\sin n\varphi-$$
$$-\left[C_n\frac{n}{r_2}J_n(r_2\sqrt{b})-C_n\sqrt{b}\,J_{n+1}(r_2\sqrt{b})+D_n\frac{n}{r_2}Y_n(r_2\sqrt{b})-\right. \tag{15}$$

$$- D_n \sqrt{b}\, Y_{n+1}(r_2 \sqrt{b}) \Big] (\cos n\varphi - 1) \Big\}. \quad (15) \text{ (cont'd)}$$

The analytical method of reducing the experimental data allows the results to be obtained in the form $\alpha = f(\varphi)$, but requires laborious calculations.

A net method [1] may also be used to solve the first two problems considered above; by this means the temperature gradient at the tube surface may be calculated using tabulated coefficients.

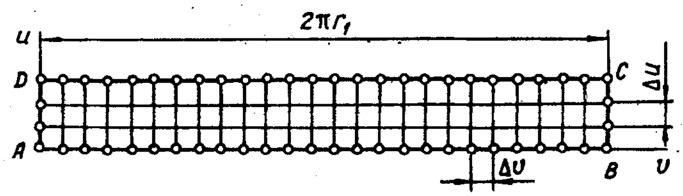

Fig. 2. Auxiliary contour for solving problems by the net method.

Introducing the new variables $u = r_1 \ln (r/r_1)$ and $v = r_1 \varphi$ used in [3] into differential equation (4), we reduce it to the form

$$\frac{\partial^2 t}{\partial u^2} + \frac{\partial^2 t}{\partial v^2} = 0. \quad (16)$$

Solution of this equation by the net method enables one to find the gradient $\partial t/\partial u$ at the surface nodes of a retangular contour (Fig. 2).

$$\frac{\partial t}{\partial u} = \frac{1}{m} \sum_{i=1}^{k} B_i t_i, \quad (17)$$

where m is the length of the contour, equal to $2\pi r_1$.

Since

$$\frac{\partial t}{\partial r} = \frac{\partial t}{\partial u} \cdot \frac{\partial u}{\partial r} \text{ and } \frac{\partial u}{\partial r} = \frac{r_1}{r},$$

$$\frac{\partial t}{\partial r} = \frac{1}{m} \frac{r_1}{r} \sum_{i=1}^{k} B_i t_i.$$

Therefore, on the outer surface of the tube

$$\left(\frac{\partial t}{\partial r} \right)_{r_2} = \frac{1}{2\pi r_2} \sum_{i=1}^{k} B_i t_i. \quad (18)$$

When the annular contour is replaced by a rectangular one, additional surfaces AD and CB (Fig. 2) appear; these correspond to one of the radial sections of the tube. To calculate the temperature gradient from (18), it is necessary to know the temperature distribution over the entire contour ABCD, including the surfaces AD and CB. However, there is no need to measure the temperature distribution in any of the radial sections of the tube.

The problem of determining the heat transfer coefficient at the tube surface by the above method is of interest only when the coefficient varies with the angle φ. Under these conditions the relations $\alpha = f(\varphi)$ and $t_2 = \zeta(\varphi)$ are characterized by at least two extrema. Conditions $\alpha_1 \neq f(\varphi)$ in which the extreme values of the functions $t_2 = \zeta(\varphi)$ and $t_1 = \xi(\varphi)$ will coincide are easy to reproduce experimentally. If we make the initial radial section $\varphi = 0$ (Fig. 1) coincide with the section where the extremum is observed, then the radial temperature distribution in this section is given by the formula for the one-dimensional problem

$$t = t_1 - \frac{t_1 - t_2}{\ln (r_2/r_1)} \ln (r/r_1). \quad (19)$$

The need to obtain a square net on the rectangular contour ABCD limits the choice of dimensions for the experimental tube.

Since

$$\Delta v = 2\pi r_1/x \text{ and } \Delta u = \ln(r_2/r_1)\; r_1/z, \tag{20}$$

the condition $\Delta v = \Delta u$ enables us to obtain

$$\frac{r_2}{r_1} = \exp\frac{2\pi z}{x}. \tag{21}$$

The network for determining the temperatures at the nodal points t_i may be superimposed directly on a drawing of the tube cross section (Fig. 1).

Since

$$\Delta\varphi = \Delta v/r_1 = \Delta u/r_1,$$

by (20) for $\Delta\varphi$ we get:

$$\Delta\varphi = \frac{1}{z}\ln\frac{r_2}{r_1}.$$

The formulas for the radii r', r", etc. are obtained from the condition that the values of Δu for adjacent sections are equal. Thus, at z = 3

$$r' = \sqrt[3]{r_2 r_1^2} \text{ and } r'' = \sqrt{r_2\sqrt[3]{r_2 r_1^2}}.$$

For a tube with heat release in the wall, when $q_v \neq f(t)$, it is also possible to solve the problem by the net method. After the change of variables $T = t + \frac{q_v}{\lambda}\frac{r^2}{4}$, $u = r_1\ln\frac{r}{r_1}$ and $v = r_1\varphi$, Eq. (9) becomes

$$\partial^2T/\partial u^2 + \partial^2T/\partial v^2 = 0. \tag{22}$$

In this case

$$\left(\frac{\partial t}{\partial r}\right)_{r_2} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial r} - \frac{q_v}{\lambda}\frac{r_2}{2}. \tag{23}$$

Taking into account the equality

$$\frac{\partial T}{\partial u} = \frac{1}{m}\sum_{i=1}^{k} B_i T_i$$

we may write (23) in the form

$$\left(\frac{\partial t}{\partial r}\right)_{r_2} = \frac{1}{2\pi r_2}\sum_{i=1}^{k} B_i T_i - \frac{q_v}{\lambda}\frac{r_2}{2}. \tag{24}$$

The position of the nodes in relation to the cross section of the tube is found in the same way as in the previous problem. The value of the T_i in (24) is determined at each node:

$$T_i = t_i + \frac{q_v}{\lambda}\frac{r^2}{4}.$$

The temperatures t_i on the curved parts of the contour are determined by direct measurement or interpolation, a and on the straight part $\varphi = 0$ by calculation from (19).

The average heat transfer coefficient is determined by graphical integration.

NOTATION

A_0, B_0, A_n, B_n, C_n, D_n – coefficients; B_i – tabulated coefficient; i – ordinal number of surface node; J_0, J_n, Y_0,

Y_n – Bessel functions; K – number of surface nodes; q_v – internal heat source output; r_1, r_2 – inside and outside tube radii; t_1, t_2 – temperatures on inside and outside tube surfaces; T – function of t; u, v – variables depending on the coordinates; x, z – number of cells in net horizontally and vertically; α_1, α – local and average heat transfer coefficients; θ – temperature difference between surrounding medium and outside surface of tube; λ – thermal conductivity of tube material; φ – angular coordinate.

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